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# Interval Estimation for the Difference between Variances of Nonnormal Distributions that Utilize the Kurtosis

[1*]**Sirima Suwan and** [1]**Sa-aat Niwitpong**

*[1]Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand*

*E-mail: sirimasuwan@hotmail.com*

**Corresponding author*

## ABSTRACT

The minimum mean-squared error-best biased estimator (MBBE) of variance as proposed by Wencheko and Chipoyera [Estimation of the variance when Kurtosis is known, Stat Papers, 50:455-464,2009] have been used in this article by adjusting kurtosis estimation procedure based on trimmed mean and to be used in the construction of two asymptotic interval estimations for the difference between two independent variances of nonnormal distributions that utilizes the kurtosis via two hybrid methods. The first hybrid method is to estimate or recover the variances of the two variance estimates which are required for constructing the confidence interval for the difference of variances from the confidence limits for the two individual variances. The second hybrid method is to construct a confidence interval for the variance difference in an analogous way as recently proposed by Herbert, Hayen, Macaskill and Walter [Interval estimation for the difference of two independent variances. Communications in Statistics, Simulation and Computation, 40:744-758, 2011]. In the case where there is no difference between population variances, simulation results shown in terms of coverage probabilities and average widths that the confidence intervals generated from these two hybrid methods can be highly recommended in asymmetric (skewed) and symmetric distributions, respectively, because they both are not only perform well in the sense that both can generally well control the coverage probabilities to be closed enough to the nominal level when sample sizes are moderate or large regardless of balance or unbalance designs but also outperform than that of their existing confidence intervals which were established from the usual unbiased sample variance estimator. However, the confidence interval produced from the first method seems to be more preferable since it also hold its level well even for symmetric distributions but are slightly liberal only for highly leptokurtic while the other is liberal for asymmetric distributions. When the difference in variances occurs and distributions are normal in shape, the confidence interval generated from the first method is the most appropriate. The simulation based on the case in which variances are unequal or observations are drawn from dissimilar nonnormal distribution shapes have already been considered but the result appears to be out of interest.

Keywords: Minimum mean– squared error, MOVER, Kurtosis, interval estimation, biased estimator.

## 1. INTRODUCTION

An improved estimator of the variance that utilizes the kurtosis was initially derived by Seals and Intarapanich (1990) and later generalized by Wencheko and Chipoyera (2007). The estimator has the form $S_W^2$ = w(n-1)$S^2$ where the weight, w = [(n+1)+( $\gamma_4$ – 3)$n^{-1}$ (n-1)]$^{-1}$ is an optimal value that minimizes the MSE ($S_W^2$) and $\gamma_4$ is the kurtosis. Wencheko *et al.* (2007) defined this estimator of $\sigma^2$ as the "minimum mean-squared error best biased estimator" (MBBE).Since the relative efficiency (RE) of the MBBE is larger than 1, thus, implying that the MBBE is always more efficient than the usual unbiased estimator $S^2$ of variance. This statistic is of interested, in the present paper, we intended to deal with the MBBE of variance by adjusting a kurtosis estimation procedure using trimmed mean (and later let's called the adjusted MBBE of variance), then making used of it to establish the confidence intervals for difference between variances. By the way, there is an alternative approach to construction of confidence intervals for the difference in variance involves using the readily available method of Zou and Donner (2008) who summarized their ideas as the Method of Variance Estimates Recovery (Mover : Zou (2008)). The MOVER combines confidence intervals based on separate samples and has identical spirits. (Qiong Li *et al*. (2011)). This method is quite convenient and effective approach for constructing confidence intervals for difference of parameters. Hence, with the $S^2$ , $S_W^2$ ,adjusted MBBE of variance and the MOVER–type confidence intervals three new interval estimations for the difference between two nonnormal population variances are desired. The comparing of these three estimator's performances is included.

As recently proposed, Herbert *et al*. (2011) had been described a simple analytical method to calculate confidence intervals for the difference of two independent samples, with reason, the methods for interval estimation have not been described before. In their investigation, the authors suggested that, at least when the observations are normally distributed with equal variances and equal sample sizes, it may be reasonable to generate confidence intervals for the difference in variances by assuming that its sampling distribution is approximately normal [Herbert et al., 2011]. In light of that, the second statistical procedure that we used to construct the confidence interval for the variance difference of nonnormal distributed population based on the adjusted MBBE of variance is then adopted Herbert et al.'s approach. This article aims to investigate how the unbiased estimator $S^2$ and the adjusted MBBE of variance perform relative to the proposed interval estimation for the variance difference procedures when data are non normal.

## 2. THE PROPOSED INTERVAL ESTIMATION PROCEDURES

Let $X_{11},...,X_{1n1},X_{21},...,X_{2n2}$ be two continuous independent samples, each sample being identical independent with distribution function $G_i(x)$, mean $\mu_i$, variance $\sigma_i^2$ and finite fourth moments $\gamma_{4i}$ for i= 1,2.The sample means and variances are $\bar{X}_i = \sum_{j=1}^{n_i} X_{ij} \Big/ n_i, i=1,2$ and $S_i^2 = \sum_{j=1}^{n_i} (X_j - \bar{X}_i)^2 \Big/ (n_i - 1), i=1,2,$ respectively. In the sections that follows, we present two hybrid methods for making inference about a confidence interval for the difference between two population variances $\sigma_1^2 - \sigma_2^2$.

### 2.1 Asymptotic Normal Distributions (General approach)

#### 2.1.1 An unbiased population variance estimator

It is well known that the usual unbiased estimate of variance is $S_i^2$, i =1,2 and its variance that available in statistical literature is given by $Var(S_i^2) = [\gamma_{4i} - (n_i - 3)/(n_i - 1)]\sigma_i^4 / n_i$ where $\gamma_{4i} = \mu_i^4 / \sigma_i^4$ and $\mu_i^4$ is the population fourth central moment. For samples sufficiently large provided the population fourth moment is finite, the sample variance is asymptotically normally distributed with mean E($S_i^2$ )and variance V($S_i^2$). A simple large-sample procedure for constructing a 100 (1-α) % confidence interval for variance can be obtained as

$$\frac{S_i^2}{1 + z_{\alpha/2}\sqrt{[\hat{\gamma}_{4i} - (n_i - 3)/(n_i - 1)]/n_i}} \le \sigma_i^2 \le \frac{S_i^2}{1 - z_{\alpha/2}\sqrt{[\hat{\gamma}_{4i} - (n_i - 3)/(n_i - 1)]/n_i}} \quad (1)$$

where $\hat{\gamma}_{4i} = \sum_{j=1}^{n_i} (X_{ji} - \bar{X}_i)^4 / n_i S_i^4$ and $z_{\alpha/2}$ be a critical z-value. Another approach is to make use of the MBBE of variance in the similar pattern of (1).

### 2.1.2 The MBBE of variance

The MBBE of variance is of the form $S_{wi}^2 = w_i(n_i - 1)S_i^2 = S_i^2 / [\{(n_i+1)/(n_i-1)\} + (\gamma_{4i} - 3)/n_i]$ and $E(S_{wi}^2) = w(n_i - 1)\sigma_i^2,\ i = 1, 2,$ where $w_i = 1/[(n_i+1) + (\gamma_{4i} - 3)(n_i - 1)/n_i],\ 0 < w_i < 1$ $MSE(S_{wi}^2) = E(S_{wi}^2 - \sigma_i^2)^2$ $= w_i^2 (n_i - 1)^2 Var(S_i^2) + [(n_i - 1)w_i - 1]^2 \sigma_i^4$, where $\gamma_{4i}$ is the kurtosis.

For large $n_i$ , when randomly sampling from any distribution with a finite fourth moment , and By the central limit theorem , The MBBE of variance is approximately standard normal with $E(S_{wi}^2)$ and $MSE(S_{wi}^2)$ .Consequently, an approximate two–sided 100 (1-α)% confidence interval for the variance may be given as

$$L = S_i^2 / \{1 + z_{\alpha/2} \sqrt{[\{\hat{\gamma}_{4i} - (n_i - 3)/(n_i - 1)\}/n_i] + [1 - 1/\hat{w}_i(n_i - 1)]^2}\},$$

$$U = S_i^2 / \{1 - z_{\alpha/2} \sqrt{[\{\hat{\gamma}_{4i} - (n_i - 3)/(n_i - 1)\}/n_i] + [1 - 1/\hat{w}_i(n_i - 1)]^2}\}, \quad (2)$$

where $\hat{\gamma}_{4i} = \sum_{j=1}^{n_i} (X_{ji} - \bar{X}_i)^4 / n_i S_i^4$, $z_{\alpha/2}$ be a critical z-value and $\hat{w}_i$ $= [(n_i + 1) + (\hat{\gamma}_{4i} - 3)(n_i - 1)/n_i]^{-1}$.

### 2.1.3 The adjusted MBBE of variance

Since an estimate of $MSE(S_{wi}^2)$ will require an estimate of kurtosis, and it is well known that a usual kurtosis estimate $\hat{\gamma}_{4i} = \sum_{j=1}^{n_i} (X_{ji} - \bar{X}_i)^4 / n_i S_i^4$, i = 1,2, was badly biased in sampling from nonnormal populations, an alternative adjusted kurtosis estimate then has been used and is of the form:

$$\hat{\gamma}_{4i}^{'} = \sum_{j=1}^{n_i} (X_{ij} - m_i)^4 / n_i S_i^4$$

where $m_i$ is a trimmed mean with trim–proportion equal to $1/2\sqrt{n_i - 4}$. Note that we used the trimmed mean in place of mean as suggested by Bonett (2006) because the trimmed mean not only tends to provide a better kurtosis estimate but also tends to improve the accuracy of the interval estimation for leptokurtic (heavy-tailed) or skewed distributions. This adjustment MBBE estimator of variance (adjusted $(\text{MBBE})_i$) yields the two sided 100(1-α) % confidence interval for variance:

$$L = S_i^2 / \{1 + z_{\alpha/2}\sqrt{[\{\hat{\gamma}_{4i}' - (n_i - 3)/(n_i - 1)\}/n_i] + [1 - 1/\hat{w}_i'(n_i - 1)]^2}\}$$

$$U = S_i^2 / \{1 - z_{\alpha/2}\sqrt{[\{\hat{\gamma}_{4i}' - (n_i - 3)/(n_i - 1)\}/n_i] + [1 - 1/\hat{w}_i'(n_i - 1)]^2}\} \quad (3)$$

where $\hat{\gamma}_{4i}' = \sum_{j=1}^{n_i}(X_{ji} - m_i)^4 / n_i S_i^4$, $z_{\alpha/2}$ be a critical z-value and $\hat{w}_i' = [(n_i + 1) + (\hat{\gamma}_{4i}' - 3)(n_i - 1)/n_i]^{-1}$.

## 2.2 The hybrid methods

Suppose we would like to construct two sided 100 (1-α) % confidence interval, denote by $(L, U)$ for $\theta_1$-$\theta_2$ where $\theta_1$, $\theta_2$ denote any two interested parameters. By the central limit theorem, if $\hat{\theta}_1$ , and $\hat{\theta}_2$ be two independent point estimates which are normally distributed, then the lower limit $L$ and the upper limit $U$ are given respectively, by

$$L = \hat{\theta}_1 - \hat{\theta}_2 - z_{\alpha/2}\sqrt{\hat{\text{var}}(\hat{\theta}_1) + \hat{\text{var}}(\hat{\theta}_2)}, \quad (4)$$

and

$$U = \hat{\theta}_1 - \hat{\theta}_2 + z_{\alpha/2}\sqrt{\hat{\text{var}}(\hat{\theta}_1) + \hat{\text{var}}(\hat{\theta}_2)}, \quad (5)$$

where $z_{\alpha/2}$ is the upper α/2- th percentile of the standard normal distribution.

However, this procedure performs well only when the sampling distributions of $\hat{\theta}_i, i = 1, 2$ are close to normal distribution or when sample sizes are sufficiently large. The analogue of the MOVER and of the study by Herbert *et al.* (2011) methods will be considered in detail and apply to be used to construct confidence intervals for variance difference.

### 2.2.1 The first method [The MOVER approach]

From equations (4) and (5), the Mover approach tries to improve confidence interval estimates by replacing the variance estimates, $\hat{var}(\hat{\theta}_i), i = 1,2$ by estimates that are in the neighborhood of the confidence limits L and U, respectively. Let ($l_1$, $u_1$) and ($l_2$, $u_2$) be separate confidence limits for $\hat{\theta}_1$ and $\hat{\theta}_2$ respectively, then ($l_1$, $u_1$) and ($l_2$, $u_2$) contain the plausible parameter values for $\hat{\theta}_1$, and $\hat{\theta}_2$ respectively. Among all these plausible values for $\hat{\theta}_1$ and $\hat{\theta}_2$ the values closest to the minimum L and the maximum U are, respectively, ($l_1$-$u_2$) and ($u_1$ –$l_2$) in the spirit of the score-type confidence interval (Bartlett (1953)).

According to Zou and Donner (2008), the variance estimates can now be recovered from $\hat{\theta}_1 = l_1$ as $\hat{var}(\hat{\theta}_1) = (\hat{\theta}_1 - l_1)^2 / z^2_{\alpha/2}$ and from $\hat{\theta}_2 = u_2$ as $\hat{var}(\hat{\theta}_2) = (u_2 - \hat{\theta}_2)^2 / z^2_{\alpha/2}$. Substituting these back into equation (4) yields

$$L = \hat{\theta}_1 - \hat{\theta}_2 - \sqrt{\left(\hat{\theta}_1 - l_1\right)^2 + \left(u_2 - \hat{\theta}_2\right)^2}. \qquad (6)$$

Similarly, the recovered from $\hat{\theta}_1 = u_1$ we have $\hat{var}(\hat{\theta}_1) = (u_1 - \hat{\theta}_1)^2 / z^2_{\alpha/2}$ and from $\hat{\theta}_2 = l_2$ we have $\hat{var}(\hat{\theta}_2) = (\hat{\theta}_2 - l_2)^2 / z^2_{\alpha/2}$. Substituting these back into equation (5) yields

$$U = \hat{\theta}_1 - \hat{\theta}_2 + \sqrt{\left(u_1 - \hat{\theta}_1\right)^2 + \left(\hat{\theta}_2 - l_2\right)^2}. \qquad (7)$$

This procedure advantage requirement is only the availability of separate confidence limits that have coverage levels close to nominal, and does not require that the distributions of $\hat{\theta}_i (i = 1,2)$ follows specific forms or to be symmetric. When the sampling distribution for $\hat{\theta}_i = (i = 1,2)$ are symmetric, it directly shows that the method leads to the conventional confidence intervals.

To obtain a confidence interval for the difference between variances via equation (6) and (7) we should have two separate confidence intervals for $\sigma_i^2$ ,i=1,2 (i.e.,( $l_1$, $u_1$)and ($l_2$, $u_2$)). Based on the three intervals of equation (1), (2) and (3), respectively, the three different hybrid confidence intervals can then be easily computed as they all have closed form solutions. Hence, the traditional MOVER limits of each for $\sigma_1^2 - \sigma_2^2$ are as follows,

(i) namely U1:

$$L = S_1^2 - S_2^2 - \sqrt{(S_1^2 - \ell_1)^2 + (u_2 - S_2^2)^2},$$
$$U = S_1^2 - S_2^2 + \sqrt{(u_1 - S_1^2)^2 + (S_2^2 - \ell_2)^2}$$

where ( $l_i$, $u_i$) , i=1,2 denote an available (1-α)100% confidence intervals for $\sigma_i^2$, i = 1,2 given by equation (1).

(ii) namely M1:

$$L = \hat{\sigma}_1^2 - \hat{\sigma}_2^2 - \sqrt{(\hat{\sigma}_1^2 - l_1)^2 + (u_2 - \hat{\sigma}_2^2)^2},$$
$$U = \hat{\sigma}_1^2 - \hat{\sigma}_2^2 - \sqrt{(u_1 - \hat{\sigma}_1^2)^2 + (\hat{\sigma}_2^2 - l_2)^2}$$

where( $l_i$, $u_i$) , i=1,2 denote an available (1-α)100% confidence intervals for $\sigma_i^2$, i = 1,2 given by equation (2) where $\hat{\sigma}_i^2 = MBBE = S_{wi}^2 = \hat{w}_i(n_i - 1)S_i^2$, $\hat{w}_i = [(n_i + 1) + (\hat{\gamma}_{4i} - 3)(n_i - 1)/n_i]^{-1}$ and $\hat{\gamma}_{4i} = \sum_{j=1}^{n_i}(X_{ji} - \bar{X}_i)^4 / n_i S_i^4$.

(iii) namely M2:

$$L = \hat{\sigma}_1^2 - \hat{\sigma}_2^2 - \sqrt{(\hat{\sigma}_1^2 - l_1)^2 + (u_2 - \hat{\sigma}_2^2)^2},$$
$$U = \hat{\sigma}_1^2 - \hat{\sigma}_2^2 - \sqrt{(u_1 - \hat{\sigma}_1^2)^2 + (\hat{\sigma}_2^2 - l_2)^2}$$

where ($l_i$, $u_i$) , i=1,2 denote an available (1-α)100% confidence intervals for $\sigma_i^2$, i = 1,2 given by equation (3) where $\hat{\sigma}_i^2 = \text{adjusted}(MBBE)_i = \hat{w}_i'(n_i - 1)S_i^2$,

$\hat{w}'_i = [(n_i + 1) + (\hat{\gamma}'_{4i} - 3)(n_i - 1)/n_i]^{-1}$, $\hat{\gamma}'_{4i} = \sum_{j=1}^{n_i} (X_{ji} - m_i)^4 / n_i S_i^4$ and $m_i$ is a trimmed mean with trim–proportion equal to $1/2\sqrt{n_i - 4}$.

### 2.2.2 The second hybrid method

In a recent study of Herbert *et al.* (2011), an interval estimation of difference between two independent variances was made by assuming that its sampling distribution is approximately normal if at least the underlying distribution of the observations are Gaussian with equal variances and equal sample sizes and their suggested confidence interval for the difference in variance is of the form:

$$S_1^2 - S_2^2 \pm z_{\frac{\alpha}{2}} \sqrt{\mathrm{Var}(S_1^2) + \mathrm{Var}(S_2^2)}$$

$$= S_1^2 - S_2^2 \pm z_{\frac{\alpha}{2}} \sqrt{S_1^4 \left[ \frac{\hat{\gamma}_{4*}}{n_1} - \frac{(n_1 - 3)}{n_1(n_1 - 1)} \right] + S_2^4 \left[ \frac{\hat{\gamma}_{4*}}{n_2} - \frac{(n_2 - 3)}{n_2(n_2 - 1)} \right]} \qquad (8)$$

where $\hat{\gamma}_{4*}$ is the Bonett's estimate of the kurtosis which is estimated by pooling the numerators and denominators of the individual Bonett's estimates of kurtosis for each group defined by

$$\hat{\gamma}_{4*} = \frac{(\sum n_i) \sum \sum (X_{ij} - m_i)^4}{[\sum \sum (X_{ij} - \bar{X}_i)^2]^4}, i = 1, 2, j = 1, \ldots n_i$$

where $m_i$ is a trimmed mean with trim proportion equal to $1/2\sqrt{n_i - 4}$ (see Herbert *et al*. (2011) for their motivation and derivation). Without loss of generality, this approach may be adapted for producing confidence interval of the variance difference based on the two asymptotic estimators of variances $S_i^2$ and the adjusted (MBBE)$_i$ (rename as adjusted ($S_{wi}^2$)). Thus, analogously, with the usual unbiased estimator $S_i^2$, we shall obtain an approximate two-sided (1-α) 100% confidence limits for $\sigma_1^2 - \sigma_2^2$ in the similar pattern:

(i) Namely U2:

$$S_1^2 - S_2^2 \pm z_{\frac{\alpha}{2}} \sqrt{S_1^4 \left[ \frac{\hat{\gamma}_{4*}}{n_1} - \frac{(n_1-3)}{n_1(n_1-1)} \right] + S_2^4 \left[ \frac{\hat{\gamma}_{4*}}{n_2} - \frac{(n_2-3)}{n_2(n_2-1)} \right]}$$

where $m_i$ is a trimmed mean with trim proportion equal to $1/2\sqrt{n_i - 4}$, $z_{\alpha/2}$ be a critical z-value and $\hat{\gamma}_{4*} = \frac{(\sum n_i)\sum\sum (X_{ij} - m_i)^4}{[\sum\sum (X_{ij} - \bar{X}_i)^2]^4}, i = 1, 2, j = 1, \ldots n_i$.

Analogously, with the two asymptotic adjustment of biased sample variances, the adjusted $(MBBE)_1$ and adjusted $(MBBE)_2$, an approximate two–sided (1-α) 100% confidence limits for $\sigma_1^2 - \sigma_2^2$ is proposed:

(ii) Namely M3:

$$\hat{\sigma}_1^2 - \hat{\sigma}_2^2 \pm z_{\alpha/2} \sqrt{M\hat{S}E(adjusted(S_{w1}^2)) + M\hat{S}E(adjusted(S_{w2}^2))}$$

$$= a_1 S_1^2 - a_2 S_2^2 \pm z_{\frac{\alpha}{2}} \sqrt{a_1^2 S_1^4 \left[ \frac{\hat{\gamma}_{4*}}{n_1} - \frac{(n_1-3)}{n_1(n_1-1)} \right] + (a_1-1)^2 S_1^4 + a_2^2 S_2^4 \left[ \frac{\hat{\gamma}_{4*}}{n_2} - \frac{(n_2-3)}{n_2(n_2-1)} \right] + (a_2-1)^2 S_2^4}$$

where

$$a_i = \hat{w}_i^*(n_i - 1) \qquad \hat{w}_i^* = [(n_i+1) + (\hat{\gamma}_{4*} - 3)(n_i - 1)/n_i]^{-1}, \qquad \hat{\gamma}_{4*} = (n_1 + n_2)$$

$$\left[\sum_{j=1}^{n_1} (X_{1j} - m_1)^4 + \sum_{j=1}^{n_2} (X_{2j} - m_2)^4\right] \Big/ \left[\sum_{j=1}^{n_1} (X_{1j} - \bar{X}_1)^2 + \sum_{j=1}^{n_2} (X_{2j} - \bar{X}_2)^2\right]^2,$$

$m_i$ is a trimmed mean with trim proportion equal to $1/2\sqrt{n_i - 4}$ and $z_{\alpha/2}$ be a critical z-value.

# 3. SIMULATION RESULTS

## 3.1 Method

A simulation study was carried out to investigate the performance of the two different methods (described in the previous section) for calculating 95% confidence limits for the difference in variances.

Estimates of the coverage probabilities (Cps) and the average interval widths (Aws) of U1, U2, M1, M2 and M3, respectively, were obtained using 50,000 pairs of two random samples of given balance and unbalance of various sample sizes from several types of distributions such as symmetric, symmetric with heavy–tailed (leptokurtic), symmetric with light–tailed (platykurtic), skewed, skewed with heavy–tailed and skewed with light–tailed distributions. The simulation programs were written in R and execute on an Intel computer.

### 3.2 Results

The performance of no difference in variances for a variety of nonnormal distributions was first investigated in term of coverage probabilities and the estimated average confidence intervals widths for the U1, M1, M2, M3 and U2 when various sample sizes are balanced and unbalanced designs and the results are summarized in Table 1. We also determined for the performances of the variance difference when samples are drawn from normal distributions in which there is a difference or no difference in variances, and then the results are shown in Table 2.

The simulation results as shown in Table1(some are not shown here) suggest that, when samples come from the symmetric nonnormal distributions with light tails (i.e., Be(3,3)), normal tails (i.e., t (10) and logit(0,1)) and heavy tails (i.e., CN (0.8, 3), t(5) and lpl (0,1)), the U2 and M3 always provide all higher estimate coverage probabilities of confidence intervals for moderate to large in both equal and unequal sample sizes but the M3 seems to be the best performer since its coverage is constantly quite close to the nominal while the M1 and M2 are regularly identical and well perform but somewhat wider than the M3 except in some light tail distribution (such as u(0,1)) that the U1 is outperform in both designs. When samples come from asymmetric (skewed) distributions with nearly normal tails (i.e., chi (5) and chi (10)), moderately heavy tails (i.e., Be (8,1) and Be (1,10)) and heavy tails (i.e., chi (3) and exp(1)), the M2 is superior since it is less sensitive than others for moderate to large in sample sizes regardless of balanced or unbalanced designs. For small sample sizes, in general, we cannot recommend any interval estimations since the investigation suggests that the fifth interval estimations often provided inaccurate coverage probabilities that either exceed or below the nominal level. Finally, when the samples come from a highly skewed distribution (i.e., ln (0,1)) with neither balanced nor unbalanced designs any of intervals investigated in this study will not be acceptable because of their too liberal performances. However, there are some evidences to ensure that all the intervals investigated tend to the target level as sample sizes are sufficiently large.

TABLE 1: Estimated coverage probabilities (Cps) and average widths (Aws) for UI, M1, M2, U2 and M3 for a variety of nonnormal distributions with no difference between variances when various sample sizes are balanced and unbalanced designs

| | | | | | | CN (.8,3) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **n1** | **n2** | **Cps (U1)** | **Aws (U1)** | **Cps (M1)** | **Aws (M1)** | **Cps (M2)** | **Aws (M2)** | **Cps (U2)** | **Aws (U2)** | **Cps (M3)** | **Aws (M3)** |
| 10 | 10 | 0.8893 | 29.71 | 0.9178 | 25.32 | 0.9404 | 46.18 | 0.9971 | 2.74 | 0.9907 | 2.46 |
| 25 | 25 | 0.9296 | 2.41 | 0.9451 | 3.26 | 0.9496 | 3.97 | 0.9749 | 1.60 | 0.9666 | 1.54 |
| 50 | 50 | 0.9424 | 1.32 | 0.9514 | 1.40 | 0.9530 | 1.42 | 0.9625 | 1.12 | 0.9581 | 1.09 |
| 100 | 100 | 0.9456 | 0.86 | 0.9499 | 0.88 | 0.9506 | 0.89 | 0.9551 | 0.79 | 0.9528 | 0.78 |
| 125 | 125 | 0.9471 | 0.75 | 0.9510 | 0.77 | 0.9519 | 0.77 | 0.9552 | 0.70 | 0.9532 | 0.70 |
| 150 | 150 | 0.9471 | 0.68 | 0.9507 | 0.69 | 0.9511 | 0.70 | 0.9539 | 0.64 | 0.9523 | 0.64 |
| 200 | 200 | 0.9484 | 0.58 | 0.9514 | 0.59 | 0.9517 | 0.59 | 0.9536 | 0.56 | 0.9524 | 0.55 |
| 250 | 250 | 0.9488 | 0.51 | 0.9506 | 0.52 | 0.9508 | 0.52 | 0.9524 | 0.50 | 0.9516 | 0.49 |
| 300 | 300 | 0.9476 | 0.47 | 0.9495 | 0.47 | 0.9499 | 0.47 | 0.9505 | 0.45 | 0.9496 | 0.45 |
| 10 | 20 | 0.8976 | 23.65 | 0.9221 | 19.75 | 0.9392 | 20.92 | 0.9655 | 2.29 | 0.9535 | 2.10 |
| 25 | 50 | 0.9305 | 1.90 | 0.9425 | 2.43 | 0.9458 | 2.36 | 0.9578 | 1.38 | 0.9514 | 1.34 |
| 50 | 100 | 0.9402 | 1.10 | 0.9468 | 1.15 | 0.9483 | 1.16 | 0.9532 | 0.97 | 0.9497 | 0.95 |
| 100 | 200 | 0.9461 | 0.73 | 0.9498 | 0.74 | 0.9504 | 0.75 | 0.9530 | 0.68 | 0.9512 | 0.67 |
| 125 | 250 | 0.9468 | 0.64 | 0.9500 | 0.65 | 0.9505 | 0.65 | 0.9513 | 0.61 | 0.9499 | 0.60 |
| 150 | 300 | 0.9469 | 0.58 | 0.9496 | 0.59 | 0.9502 | 0.59 | 0.9511 | 0.56 | 0.9495 | 0.55 |
| 200 | 400 | 0.9477 | 0.50 | 0.9495 | 0.50 | 0.9499 | 0.50 | 0.9512 | 0.48 | 0.9504 | 0.48 |
| 250 | 500 | 0.9481 | 0.44 | 0.9499 | 0.45 | 0.9500 | 0.45 | 0.9512 | 0.43 | 0.9506 | 0.43 |
| 300 | 600 | 0.9479 | 0.40 | 0.9491 | 0.40 | 0.9494 | 0.40 | 0.9504 | 0.39 | 0.9496 | 0.39 |
| 20 | 10 | 0.9003 | 12.77 | 0.9244 | 16.20 | 0.9412 | 31.36 | 0.9649 | 2.29 | 0.9524 | 2.10 |
| 50 | 25 | 0.9315 | 1.90 | 0.9438 | 2.27 | 0.9471 | 3.08 | 0.9593 | 1.38 | 0.9529 | 1.33 |
| 100 | 50 | 0.9401 | 1.10 | 0.9469 | 1.15 | 0.9482 | 1.16 | 0.9547 | 0.97 | 0.9512 | 0.95 |
| 200 | 100 | 0.9457 | 0.73 | 0.9497 | 0.74 | 0.9501 | 0.75 | 0.9523 | 0.68 | 0.9506 | 0.67 |
| 250 | 125 | 0.9455 | 0.64 | 0.9488 | 0.65 | 0.9494 | 0.65 | 0.9503 | 0.61 | 0.9485 | 0.60 |
| 300 | 150 | 0.9479 | 0.58 | 0.9502 | 0.59 | 0.9507 | 0.59 | 0.9520 | 0.56 | 0.9506 | 0.55 |
| 400 | 200 | 0.9472 | 0.50 | 0.9491 | 0.50 | 0.9494 | 0.50 | 0.9510 | 0.48 | 0.9500 | 0.48 |
| 500 | 250 | 0.9468 | 0.44 | 0.9482 | 0.45 | 0.9485 | 0.45 | 0.9494 | 0.43 | 0.9487 | 0.43 |
| 600 | 300 | 0.9474 | 0.40 | 0.9488 | 0.40 | 0.9491 | 0.40 | 0.9496 | 0.39 | 0.9491 | 0.39 |
| | | | | | | **Be(8,1)** | | | | | |
| **n1** | **n2** | **Cps(U1)** | **Aws(U1)** | **Cps(M1)** | **Aws(M1)** | **Cps(M2)** | **Aws(M2)** | **Cps(U2)** | **Aws(U2)** | **Cps(M3)** | **Aws(M3)** |
| 10 | 10 | 0.7887 | 0.58 | 0.8291 | 0.42 | 0.8830 | 0.53 | 0.9939 | 0.04 | 0.9863 | 0.03 |
| 25 | 25 | 0.8727 | 0.18 | 0.8982 | 0.21 | 0.9266 | 0.30 | 0.9850 | 0.02 | 0.9773 | 0.02 |
| 50 | 50 | 0.9075 | 0.04 | 0.9233 | 0.05 | 0.9444 | 0.07 | 0.9786 | 0.02 | 0.9725 | 0.02 |
| 100 | 100 | 0.9301 | 0.01 | 0.9382 | 0.01 | 0.9548 | 0.02 | 0.9722 | 0.01 | 0.9687 | 0.01 |
| 125 | 125 | 0.9336 | 0.01 | 0.9405 | 0.01 | 0.9543 | 0.01 | 0.9682 | 0.01 | 0.9646 | 0.01 |
| 150 | 150 | 0.9352 | 0.01 | 0.9413 | 0.01 | 0.9541 | 0.01 | 0.9668 | 0.01 | 0.9641 | 0.01 |
| 200 | 200 | 0.9391 | 0.01 | 0.9440 | 0.01 | 0.9544 | 0.01 | 0.9632 | 0.01 | 0.9611 | 0.01 |
| 250 | 250 | 0.9412 | 0.01 | 0.9457 | 0.01 | 0.9552 | 0.01 | 0.9630 | 0.01 | 0.9613 | 0.01 |
| 300 | 300 | 0.9434 | 0.01 | 0.9470 | 0.01 | 0.9564 | 0.01 | 0.9618 | 0.01 | 0.9599 | 0.01 |
| 10 | 20 | 0.8142 | 0.58 | 0.8511 | 0.37 | 0.8982 | 5.20 | 0.9732 | 0.03 | 0.9591 | 0.03 |
| 25 | 50 | 0.8832 | 0.10 | 0.9046 | 0.14 | 0.9304 | 0.28 | 0.9672 | 0.02 | 0.9581 | 0.02 |
| 50 | 100 | 0.9127 | 0.02 | 0.9260 | 0.04 | 0.9454 | 0.04 | 0.9657 | 0.01 | 0.9605 | 0.01 |
| 100 | 200 | 0.9320 | 0.01 | 0.9392 | 0.01 | 0.9541 | 0.01 | 0.9636 | 0.01 | 0.9607 | 0.01 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 125 | 250 | 0.9343 | 0.01 | 0.9406 | 0.01 | 0.9534 | 0.01 | 0.9615 | 0.01 | 0.9586 | 0.01 |
| 150 | 300 | 0.9341 | 0.01 | 0.9391 | 0.01 | 0.9514 | 0.01 | 0.9601 | 0.01 | 0.9577 | 0.01 |
| 200 | 400 | 0.9383 | 0.01 | 0.9416 | 0.01 | 0.9529 | 0.01 | 0.9592 | 0.01 | 0.9572 | 0.01 |
| 250 | 500 | 0.9411 | 0.01 | 0.9445 | 0.01 | 0.9541 | 0.01 | 0.9589 | 0.01 | 0.9573 | 0.01 |
| 300 | 600 | 0.9433 | 0.01 | 0.9457 | 0.01 | 0.9545 | 0.01 | 0.9587 | 0.01 | 0.9575 | 0.01 |
| 20 | 10 | 0.8112 | 0.31 | 0.8494 | 0.47 | 0.8965 | 1.67 | 0.9719 | 0.03 | 0.9587 | 0.03 |
| 50 | 25 | 0.8842 | 0.11 | 0.9058 | 0.12 | 0.9321 | 0.18 | 0.9678 | 0.02 | 0.9584 | 0.02 |
| 100 | 50 | 0.9132 | 0.02 | 0.9259 | 0.03 | 0.9455 | 0.05 | 0.9652 | 0.01 | 0.9599 | 0.01 |

| | | Be(8,1) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n1 | n2 | Cps(U1) | Aws(U1) | Cps(M1) | Aws(M1) | Cps(M2) | Aws(M2) | Cps(U2) | Aws(U2) | Cps(M3) | Aws(M3) |
| 200 | 100 | 0.9325 | 0.01 | 0.9396 | 0.01 | 0.9536 | 0.01 | 0.9634 | 0.01 | 0.9601 | 0.01 |
| 250 | 125 | 0.9355 | 0.01 | 0.9412 | 0.01 | 0.9528 | 0.01 | 0.9619 | 0.01 | 0.9591 | 0.01 |
| 300 | 150 | 0.9374 | 0.01 | 0.9422 | 0.01 | 0.9537 | 0.01 | 0.9610 | 0.01 | 0.9588 | 0.01 |
| 400 | 200 | 0.9378 | 0.01 | 0.9418 | 0.01 | 0.9526 | 0.01 | 0.9589 | 0.01 | 0.9569 | 0.01 |
| 500 | 250 | 0.9413 | 0.01 | 0.9439 | 0.01 | 0.9531 | 0.01 | 0.9577 | 0.01 | 0.9563 | 0.01 |
| 600 | 300 | 0.9438 | 0.01 | 0.9463 | 0.01 | 0.9541 | 0.01 | 0.9587 | 0.01 | 0.9574 | 0.01 |

| | | u(0,1) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n1 | n2 | Cps(U1) | Aws(U1) | Cps(M1) | Aws(M1) | Cps(M2) | Aws(M2) | Cps(U2) | Aws(U2) | Cps(M3) | Aws(M3) |
| 10 | 10 | 0.9427 | 0.41 | 0.9555 | 0.64 | 0.9704 | 2.21 | 0.9914 | 0.18 | 0.9846 | 0.17 |
| 25 | 25 | 0.9625 | 0.10 | 0.9692 | 0.11 | 0.9723 | 0.11 | 0.9672 | 0.09 | 0.9633 | 0.09 |
| 50 | 50 | 0.9605 | 0.06 | 0.9645 | 0.07 | 0.9661 | 0.07 | 0.9602 | 0.06 | 0.9586 | 0.06 |
| 100 | 100 | 0.9562 | 0.04 | 0.9585 | 0.04 | 0.9592 | 0.04 | 0.9560 | 0.04 | 0.9551 | 0.04 |
| 125 | 125 | 0.9560 | 0.04 | 0.9572 | 0.04 | 0.9578 | 0.04 | 0.9555 | 0.04 | 0.9548 | 0.04 |
| 150 | 150 | 0.9539 | 0.03 | 0.9551 | 0.04 | 0.9555 | 0.04 | 0.9535 | 0.03 | 0.9529 | 0.03 |
| 200 | 200 | 0.9546 | 0.03 | 0.9557 | 0.03 | 0.9560 | 0.03 | 0.9542 | 0.03 | 0.9538 | 0.03 |
| 250 | 250 | 0.9515 | 0.03 | 0.9525 | 0.03 | 0.9527 | 0.03 | 0.9514 | 0.03 | 0.9511 | 0.03 |
| 300 | 300 | 0.9510 | 0.02 | 0.9520 | 0.02 | 0.9522 | 0.02 | 0.9506 | 0.02 | 0.9503 | 0.02 |
| 10 | 20 | 0.9436 | 0.26 | 0.9552 | 0.30 | 0.9661 | 0.96 | 0.9601 | 0.14 | 0.9536 | 0.14 |
| 25 | 50 | 0.9550 | 0.08 | 0.9606 | 0.09 | 0.9633 | 0.09 | 0.9564 | 0.08 | 0.9531 | 0.08 |
| 50 | 100 | 0.9536 | 0.05 | 0.9565 | 0.06 | 0.9579 | 0.06 | 0.9555 | 0.05 | 0.9540 | 0.05 |
| 100 | 200 | 0.9536 | 0.04 | 0.9552 | 0.04 | 0.9559 | 0.04 | 0.9528 | 0.04 | 0.9520 | 0.04 |
| 125 | 250 | 0.9511 | 0.03 | 0.9525 | 0.03 | 0.9528 | 0.03 | 0.9513 | 0.03 | 0.9506 | 0.03 |
| 150 | 300 | 0.9528 | 0.03 | 0.9539 | 0.03 | 0.9542 | 0.03 | 0.9520 | 0.03 | 0.9516 | 0.03 |
| 200 | 400 | 0.9519 | 0.03 | 0.9528 | 0.03 | 0.9530 | 0.03 | 0.9512 | 0.03 | 0.9508 | 0.03 |
| 250 | 500 | 0.9512 | 0.02 | 0.9519 | 0.02 | 0.9521 | 0.02 | 0.9515 | 0.02 | 0.9512 | 0.02 |
| 300 | 600 | 0.9503 | 0.02 | 0.9508 | 0.02 | 0.9509 | 0.02 | 0.9511 | 0.02 | 0.9508 | 0.02 |
| 20 | 10 | 0.9428 | 0.24 | 0.9538 | 0.43 | 0.9661 | 1.48 | 0.9615 | 0.14 | 0.9553 | 0.14 |
| 50 | 25 | 0.9556 | 0.08 | 0.9612 | 0.09 | 0.9637 | 0.09 | 0.9576 | 0.08 | 0.9549 | 0.08 |
| 100 | 50 | 0.9547 | 0.05 | 0.9574 | 0.06 | 0.9584 | 0.06 | 0.9550 | 0.05 | 0.9534 | 0.05 |
| 200 | 100 | 0.9529 | 0.04 | 0.9545 | 0.04 | 0.9550 | 0.04 | 0.9522 | 0.04 | 0.9514 | 0.04 |
| 250 | 125 | 0.9528 | 0.03 | 0.9539 | 0.03 | 0.9543 | 0.03 | 0.9517 | 0.03 | 0.9510 | 0.03 |
| 300 | 150 | 0.9526 | 0.03 | 0.9538 | 0.03 | 0.9540 | 0.03 | 0.9528 | 0.03 | 0.9524 | 0.03 |
| 400 | 200 | 0.9512 | 0.03 | 0.9520 | 0.03 | 0.9522 | 0.03 | 0.9507 | 0.03 | 0.9503 | 0.03 |
| 500 | 250 | 0.9528 | 0.02 | 0.9533 | 0.02 | 0.9535 | 0.02 | 0.9521 | 0.02 | 0.9518 | 0.02 |
| 600 | 300 | 0.9504 | 0.02 | 0.9510 | 0.02 | 0.9511 | 0.02 | 0.9512 | 0.02 | 0.9509 | 0.02 |

| | | exp(1) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n1 | n2 | Cps(U1) | Aws(U1) | Cps(M1) | Aws(M1) | Cps(M2) | Aws(M2) | Cps(U2) | Aws(U2) | Cps(M3) | Aws(M3) |
| 10 | 10 | 0.7462 | 54.5 | 0.7911 | 51.7 | 0.8486 | 31.6 | 0.9953 | 4.8 | 0.9884 | 3.7 |
| 25 | 25 | 0.8374 | 28.3 | 0.8744 | 33.3 | 0.9108 | 66.7 | 0.9894 | 3.0 | 0.9825 | 2.6 |
| 50 | 50 | 0.8792 | 16.9 | 0.9054 | 16.7 | 0.9321 | 24.1 | 0.9862 | 2.2 | 0.9802 | 2.0 |
| 100 | 100 | 0.9062 | 3.5 | 0.9218 | 4.8 | 0.9416 | 7.3 | 0.9801 | 1.6 | 0.9744 | 1.5 |
| 125 | 125 | 0.9133 | 2.7 | 0.9270 | 3.6 | 0.9434 | 4.0 | 0.9764 | 1.4 | 0.9717 | 1.4 |

| n1 | n2 | Cps(U1) | Aws(U1) | Cps(M1) | Aws(M1) | Cps(M2) | Aws(M2) | Cps(U2) | Aws(U2) | Cps(M3) | Aws(M3) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 150 | 150 | 0.9152 | 1.8 | 0.9265 | 3.7 | 0.9422 | 2.7 | 0.9745 | 1.3 | 0.9707 | 1.3 |
| 200 | 200 | 0.9231 | 1.4 | 0.9320 | 1.5 | 0.9457 | 1.9 | 0.9713 | 1.1 | 0.9679 | 1.1 |
| 250 | 250 | 0.9262 | 1.2 | 0.9338 | 1.3 | 0.9454 | 1.3 | 0.9679 | 1.0 | 0.9641 | 1.0 |
| 300 | 300 | 0.9303 | 1.0 | 0.9365 | 1.1 | 0.9479 | 1.2 | 0.9671 | 0.9 | 0.9646 | 0.9 |
| | | | | | | | | | | | |
| 10 | 20 | 0.7770 | 36.8 | 0.8203 | 157.1 | 0.8721 | 48.1 | 0.9789 | 4.2 | 0.9663 | 3.3 |
| 25 | 50 | 0.8528 | 21.7 | 0.8847 | 26.4 | 0.9174 | 92.6 | 0.9748 | 2.7 | 0.9645 | 2.4 |
| 50 | 100 | 0.8891 | 10.9 | 0.9102 | 12.9 | 0.9334 | 14.9 | 0.9730 | 1.9 | 0.9660 | 1.8 |
| 100 | 200 | 0.9095 | 2.4 | 0.9220 | 3.3 | 0.9409 | 4.4 | 0.9683 | 1.4 | 0.9635 | 1.3 |
| 125 | 250 | 0.9158 | 2.0 | 0.9258 | 2.3 | 0.9421 | 2.8 | 0.9672 | 1.2 | 0.9628 | 1.2 |
| 150 | 300 | 0.9181 | 1.5 | 0.9274 | 1.7 | 0.9415 | 2.0 | 0.9659 | 1.1 | 0.9622 | 1.1 |
| 200 | 400 | 0.9275 | 1.1 | 0.9334 | 1.3 | 0.9455 | 2.1 | 0.9642 | 1.0 | 0.9617 | 1.0 |
| 250 | 500 | 0.9303 | 1.0 | 0.9359 | 1.0 | 0.9460 | 1.1 | 0.9626 | 0.9 | 0.9598 | 0.9 |
| 300 | 600 | 0.9326 | 0.9 | 0.9369 | 0.9 | 0.9460 | 0.9 | 0.9616 | 0.8 | 0.9593 | 0.8 |

| | | | | | | **exp(1)** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n1 | n2 | **Cps(U1)** | **Aws(U1)** | **Cps(M1)** | **Aws(M1)** | **Cps(M2)** | **Aws(M2)** | **Cps(U2)** | **Aws(U2)** | **Cps(M3)** | **Aws(M3)** |
| 20 | 10 | 0.7750 | 70.6 | 0.8176 | 38.9 | 0.8691 | 31.6 | 0.9787 | 4.2 | 0.9656 | 3.3 |
| 50 | 25 | 0.8534 | 21.0 | 0.8853 | 29.5 | 0.9182 | 31.1 | 0.9748 | 2.7 | 0.9656 | 2.4 |
| 100 | 50 | 0.8907 | 10.3 | 0.9112 | 11.4 | 0.9346 | 13.0 | 0.9734 | 1.9 | 0.9667 | 1.8 |
| 200 | 100 | 0.9113 | 3.0 | 0.9234 | 4.2 | 0.9415 | 5.2 | 0.9689 | 1.4 | 0.9644 | 1.3 |
| 250 | 125 | 0.9151 | 2.0 | 0.9264 | 2.0 | 0.9418 | 2.4 | 0.9671 | 1.2 | 0.9633 | 1.2 |
| 300 | 150 | 0.9205 | 1.5 | 0.9295 | 1.8 | 0.9433 | 2.9 | 0.9649 | 1.1 | 0.9610 | 1.1 |
| 400 | 200 | 0.9251 | 1.1 | 0.9325 | 1.2 | 0.9447 | 1.3 | 0.9649 | 1.0 | 0.9617 | 1.0 |
| 500 | 250 | 0.9303 | 1.0 | 0.9356 | 1.0 | 0.9466 | 1.1 | 0.9643 | 0.9 | 0.9617 | 0.9 |
| 600 | 300 | 0.9313 | 0.9 | 0.9359 | 0.9 | 0.9459 | 0.9 | 0.9616 | 0.8 | 0.9593 | 0.8 |

| | | | | | | **chi(1)** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n1 | n2 | **Cps(U1)** | **Aws(U1)** | **Cps(M1)** | **Aws(M1)** | **Cps(M2)** | **Aws(M2)** | **Cps(U2)** | **Aws(U2)** | **Cps(M3)** | **Aws(M3)** |
| 10 | 10 | 0.7129 | 84.8 | 0.7412 | 217.6 | 0.7704 | 50.9 | 0.9965 | 12.0 | 0.9919 | 8.5 |
| 25 | 25 | 0.8332 | 160.5 | 0.8727 | 102.3 | 0.9156 | 245.5 | 0.9954 | 7.8 | 0.9902 | 6.4 |
| 50 | 50 | 0.8761 | 78.9 | 0.9089 | 70.2 | 0.9409 | 157.7 | 0.9928 | 5.7 | 0.9872 | 5.0 |
| 100 | 100 | 0.8973 | 23.2 | 0.9200 | 33.3 | 0.9446 | 51.4 | 0.9880 | 4.1 | 0.9814 | 3.8 |
| 125 | 125 | 0.9052 | 16.0 | 0.9248 | 21.8 | 0.9454 | 23.1 | 0.9850 | 3.7 | 0.9786 | 3.5 |
| 150 | 150 | 0.9082 | 12.2 | 0.9257 | 22.2 | 0.9446 | 21.2 | 0.9815 | 3.4 | 0.9753 | 3.2 |
| 200 | 200 | 0.9148 | 54.2 | 0.9292 | 7.8 | 0.9465 | 10.2 | 0.9787 | 2.9 | 0.9727 | 2.8 |
| 250 | 250 | 0.9214 | 4.0 | 0.9327 | 5.6 | 0.9470 | 6.4 | 0.9747 | 2.6 | 0.9694 | 2.5 |
| 300 | 300 | 0.9240 | 3.3 | 0.9342 | 4.0 | 0.9468 | 4.1 | 0.9726 | 2.4 | 0.9686 | 2.3 |
| | | | | | | | | | | | |
| 10 | 20 | 0.7530 | 91.6 | 0.7870 | 181.7 | 0.8192 | 59.3 | 0.9866 | 10.6 | 0.9758 | 7.8 |
| 25 | 50 | 0.8474 | 82.3 | 0.8845 | 116.1 | 0.9243 | 117.4 | 0.9821 | 6.9 | 0.9698 | 5.8 |
| 50 | 100 | 0.8810 | 65.1 | 0.9107 | 58.6 | 0.9396 | 118.4 | 0.9781 | 5.0 | 0.9683 | 4.5 |
| 100 | 200 | 0.9043 | 15.1 | 0.9232 | 370.9 | 0.9440 | 25.6 | 0.9723 | 3.6 | 0.9648 | 3.4 |
| 125 | 250 | 0.9099 | 9.4 | 0.9249 | 13.5 | 0.9417 | 15.6 | 0.9699 | 3.2 | 0.9633 | 3.0 |
| 150 | 300 | 0.9139 | 6.0 | 0.9280 | 8.6 | 0.9450 | 9.8 | 0.9705 | 2.9 | 0.9644 | 2.8 |
| 200 | 400 | 0.9166 | 4.2 | 0.9280 | 7.4 | 0.9428 | 5.8 | 0.9673 | 2.6 | 0.9625 | 2.5 |
| 250 | 500 | 0.9233 | 3.0 | 0.9328 | 3.8 | 0.9442 | 4.3 | 0.9656 | 2.3 | 0.9613 | 2.2 |
| 300 | 600 | 0.9258 | 2.6 | 0.9336 | 2.9 | 0.9451 | 3.1 | 0.9649 | 2.1 | 0.9611 | 2.0 |
| | | | | | | | | | | | |
| 20 | 10 | 0.7500 | 113.2 | 0.7849 | 79.4 | 0.8174 | 132.8 | 0.9864 | 10.6 | 0.9748 | 7.8 |
| 50 | 25 | 0.8464 | 99.0 | 0.8845 | 92.2 | 0.9244 | 88.7 | 0.9815 | 6.8 | 0.9705 | 5.7 |
| 100 | 50 | 0.8830 | 46.6 | 0.9125 | 58.9 | 0.9423 | 89.2 | 0.9774 | 5.0 | 0.9677 | 4.5 |
| 200 | 100 | 0.9019 | 14.3 | 0.9217 | 25.8 | 0.9445 | 33.0 | 0.9730 | 3.6 | 0.9662 | 3.4 |
| 250 | 125 | 0.9080 | 10.5 | 0.9247 | 18.2 | 0.9429 | 14.6 | 0.9707 | 3.2 | 0.9646 | 3.0 |
| 300 | 150 | 0.9123 | 11.6 | 0.9267 | 9.0 | 0.9437 | 10.6 | 0.9698 | 2.9 | 0.9647 | 2.8 |
| 400 | 200 | 0.9202 | 5.3 | 0.9314 | 6.4 | 0.9459 | 5.8 | 0.9673 | 2.6 | 0.9629 | 2.5 |
| 500 | 250 | 0.9240 | 3.1 | 0.9330 | 3.6 | 0.9451 | 3.6 | 0.9662 | 2.3 | 0.9620 | 2.2 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 600 | 300 | 0.9260 | 2.7 | 0.9342 | 2.9 | 0.9456 | 3.1 | 0.9649 | 2.1 | 0.9609 | 2.0 |
| | | | | | | **chi(10)** | | | | | |
| n1 | n2 | **Cps(U1)** | **Aws(U1)** | **Cps(M1)** | **Aws(M1)** | **Cps(M2)** | **Aws(M2)** | **Cps(U2)** | **Aws(U2)** | **Cps(M3)** | **Aws(M3)** |
| 10 | 10 | 0.8455 | 575.4 | 0.8848 | 840.4 | 0.9201 | 997.8 | 0.9956 | 65.1 | 0.9891 | 55.7 |
| 25 | 25 | 0.8913 | 288.7 | 0.9139 | 227.8 | 0.9265 | 704.9 | 0.9794 | 39.4 | 0.9715 | 36.8 |
| 50 | 50 | 0.9126 | 75.9 | 0.9258 | 77.5 | 0.9349 | 76.8 | 0.9711 | 28.1 | 0.9652 | 27.1 |
| 100 | 100 | 0.9265 | 22.9 | 0.9339 | 24.0 | 0.9425 | 25.5 | 0.9647 | 20.0 | 0.9612 | 19.7 |
| 125 | 125 | 0.9313 | 19.8 | 0.9372 | 20.6 | 0.9443 | 21.6 | 0.9626 | 17.9 | 0.9599 | 17.7 |
| 150 | 150 | 0.9340 | 17.8 | 0.9389 | 18.3 | 0.9456 | 19.1 | 0.9626 | 16.4 | 0.9603 | 16.2 |
| 200 | 200 | 0.9363 | 15.0 | 0.9400 | 15.3 | 0.9454 | 15.9 | 0.9595 | 14.2 | 0.9575 | 14.1 |
| 250 | 250 | 0.9371 | 13.2 | 0.9407 | 13.5 | 0.9461 | 13.8 | 0.9564 | 12.7 | 0.9552 | 12.6 |
| 300 | 300 | 0.9399 | 12.0 | 0.9426 | 12.1 | 0.9475 | 12.5 | 0.9567 | 11.6 | 0.9554 | 11.5 |
| 10 | 20 | 0.8555 | 453.0 | 0.8899 | 506.6 | 0.9188 | 815.5 | 0.9685 | 55.2 | 0.9562 | 48.5 |
| 25 | 50 | 0.8988 | 189.0 | 0.9173 | 145.6 | 0.9285 | 180.5 | 0.9645 | 34.4 | 0.9567 | 32.5 |
| 50 | 100 | 0.9180 | 34.6 | 0.9287 | 39.4 | 0.9375 | 53.0 | 0.9624 | 24.5 | 0.9575 | 23.8 |
| 100 | 200 | 0.9298 | 19.0 | 0.9355 | 23.8 | 0.9426 | 21.0 | 0.9601 | 17.4 | 0.9571 | 17.1 |
| 125 | 250 | 0.9328 | 16.7 | 0.9371 | 17.2 | 0.9433 | 17.8 | 0.9577 | 15.5 | 0.9553 | 15.4 |
| 150 | 300 | 0.9365 | 15.0 | 0.9399 | 15.4 | 0.9459 | 15.9 | 0.9582 | 14.2 | 0.9562 | 14.1 |
| | | | | | | **chi(10)** | | | | | |
| n1 | n2 | **Cps(U1)** | **Aws(U1)** | **Cps(M1)** | **Aws(M1)** | **Cps(M2)** | **Aws(M2)** | **Cps(U2)** | **Aws(U2)** | **Cps(M3)** | **Aws(M3)** |
| 200 | 400 | 0.9360 | 12.7 | 0.9392 | 13.0 | 0.9449 | 13.3 | 0.9558 | 12.3 | 0.9541 | 12.2 |
| 250 | 500 | 0.9401 | 11.3 | 0.9424 | 11.4 | 0.9477 | 11.7 | 0.9560 | 11.0 | 0.9549 | 10.9 |
| 300 | 600 | 0.9424 | 10.2 | 0.9440 | 10.4 | 0.9492 | 10.6 | 0.9563 | 10.0 | 0.9552 | 10.0 |
| 20 | 10 | 0.8557 | 431.7 | 0.8897 | 445.7 | 0.9183 | 565.0 | 0.9699 | 55.4 | 0.9589 | 48.7 |
| 50 | 25 | 0.8989 | 122.7 | 0.9174 | 220.5 | 0.9293 | 568.4 | 0.9655 | 34.3 | 0.9579 | 32.5 |
| 100 | 50 | 0.9165 | 63.8 | 0.9271 | 59.3 | 0.9359 | 120.1 | 0.9616 | 24.5 | 0.9569 | 23.8 |
| 200 | 100 | 0.9289 | 19.1 | 0.9347 | 19.9 | 0.9421 | 21.5 | 0.9587 | 17.4 | 0.9559 | 17.1 |
| 250 | 125 | 0.9327 | 16.7 | 0.9375 | 17.2 | 0.9432 | 17.9 | 0.9583 | 15.6 | 0.9562 | 15.4 |
| 300 | 150 | 0.9353 | 15.0 | 0.9392 | 15.3 | 0.9458 | 15.9 | 0.9571 | 14.2 | 0.9552 | 14.0 |
| 400 | 200 | 0.9386 | 12.7 | 0.9414 | 12.9 | 0.9470 | 13.3 | 0.9573 | 12.3 | 0.9559 | 12.2 |
| 500 | 250 | 0.9405 | 11.3 | 0.9430 | 11.5 | 0.9477 | 11.7 | 0.9556 | 11.0 | 0.9545 | 10.9 |
| 600 | 300 | 0.9426 | 10.2 | 0.9445 | 10.4 | 0.9488 | 10.6 | 0.9561 | 10.0 | 0.9553 | 10.0 |
| | | | | | | **logit(0, 1)** | | | | | |
| n1 | n2 | **Cps(U1)** | **Aws(U1)** | **Cps(M1)** | **Aws(M1)** | **Cps(M2)** | **Aws(M2)** | **Cps(U2)** | **Aws(U2)** | **Cps(M3)** | **Aws(M3)** |
| 10 | 10 | 0.8574 | 196.2 | 0.8937 | 112.3 | 0.9233 | 160.2 | 0.9976 | 10.4 | 0.9922 | 9.0 |
| 25 | 25 | 0.9040 | 26.6 | 0.9269 | 29.5 | 0.9338 | 50.9 | 0.9814 | 6.4 | 0.9722 | 6.0 |
| 50 | 50 | 0.9257 | 6.9 | 0.9392 | 8.3 | 0.9420 | 9.9 | 0.9710 | 4.5 | 0.9647 | 4.4 |
| 100 | 100 | 0.9344 | 3.8 | 0.9421 | 4.0 | 0.9439 | 4.0 | 0.9621 | 3.2 | 0.9588 | 3.2 |
| 125 | 125 | 0.9373 | 3.3 | 0.9434 | 3.4 | 0.9442 | 3.4 | 0.9586 | 2.9 | 0.9555 | 2.8 |
| 150 | 150 | 0.9408 | 2.9 | 0.9460 | 3.0 | 0.9468 | 3.0 | 0.9590 | 2.6 | 0.9569 | 2.6 |
| 200 | 200 | 0.9393 | 2.5 | 0.9432 | 2.5 | 0.9439 | 2.5 | 0.9541 | 2.3 | 0.9523 | 2.3 |
| 250 | 250 | 0.9412 | 2.2 | 0.9447 | 2.2 | 0.9450 | 2.2 | 0.9541 | 2.0 | 0.9524 | 2.0 |
| 300 | 300 | 0.9434 | 2.0 | 0.9462 | 2.0 | 0.9465 | 2.0 | 0.9536 | 1.9 | 0.9522 | 1.9 |
| 10 | 20 | 0.8705 | 72.3 | 0.9031 | 80.4 | 0.9235 | 281.6 | 0.9719 | 8.9 | 0.9580 | 7.9 |
| 25 | 50 | 0.9094 | 15.6 | 0.9273 | 20.9 | 0.9324 | 28.8 | 0.9625 | 5.5 | 0.9548 | 5.3 |
| 50 | 100 | 0.9263 | 5.4 | 0.9363 | 7.1 | 0.9383 | 7.5 | 0.9587 | 3.9 | 0.9539 | 3.8 |
| 100 | 200 | 0.9372 | 3.1 | 0.9428 | 3.3 | 0.9440 | 3.3 | 0.9557 | 2.8 | 0.9524 | 2.8 |
| 125 | 250 | 0.9377 | 2.7 | 0.9423 | 2.8 | 0.9432 | 2.8 | 0.9540 | 2.5 | 0.9516 | 2.5 |
| 150 | 300 | 0.9399 | 2.5 | 0.9437 | 2.5 | 0.9445 | 2.5 | 0.9523 | 2.3 | 0.9504 | 2.3 |
| 200 | 400 | 0.9430 | 2.1 | 0.9458 | 2.1 | 0.9463 | 2.1 | 0.9531 | 2.0 | 0.9519 | 2.0 |
| 250 | 500 | 0.9440 | 1.9 | 0.9465 | 1.9 | 0.9470 | 1.9 | 0.9530 | 1.8 | 0.9520 | 1.8 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 300 | 600 | 0.9434 | 1.7 | 0.9460 | 1.7 | 0.9463 | 1.7 | 0.9511 | 1.6 | 0.9499 | 1.6 |
| 20 | 10 | 0.8688 | 161.4 | 0.9020 | 153.2 | 0.9224 | 108.3 | 0.9719 | 8.9 | 0.9584 | 7.9 |
| 50 | 25 | 0.9122 | 18.7 | 0.9303 | 19.5 | 0.9357 | 21.6 | 0.9629 | 5.5 | 0.9553 | 5.3 |
| 100 | 50 | 0.9255 | 5.3 | 0.9356 | 6.0 | 0.9379 | 6.7 | 0.9591 | 3.9 | 0.9534 | 3.8 |
| 200 | 100 | 0.9343 | 3.1 | 0.9403 | 3.3 | 0.9415 | 3.3 | 0.9549 | 2.8 | 0.9519 | 2.8 |
| 250 | 125 | 0.9376 | 2.7 | 0.9421 | 2.8 | 0.9430 | 2.8 | 0.9546 | 2.5 | 0.9520 | 2.5 |
| 300 | 150 | 0.9396 | 2.5 | 0.9432 | 2.5 | 0.9438 | 2.5 | 0.9538 | 2.3 | 0.9520 | 2.3 |
| 400 | 200 | 0.9422 | 2.1 | 0.9453 | 2.1 | 0.9457 | 2.1 | 0.9523 | 2.0 | 0.9507 | 2.0 |
| 500 | 250 | 0.9435 | 1.9 | 0.9461 | 1.9 | 0.9465 | 1.9 | 0.9521 | 1.8 | 0.9508 | 1.8 |
| 600 | 300 | 0.9436 | 1.7 | 0.9453 | 1.7 | 0.9456 | 1.7 | 0.9514 | 1.6 | 0.9504 | 1.6 |

| | | t(10) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n1 | n2 | **Cps(U1)** | **Aws(U1)** | **Cps(M1)** | **Aws(M1)** | **Cps(M2)** | **Aws(M2)** | **Cps(U2)** | **Aws(U2)** | **Cps(M3)** | **Aws(M3)** |
| 10 | 10 | 0.8641 | 36.5 | 0.9002 | 58.8 | 0.9271 | 89.5 | 0.9974 | 3.9 | 0.9927 | 3.4 |
| 25 | 25 | 0.9115 | 13.3 | 0.9310 | 10.6 | 0.9374 | 14.6 | 0.9815 | 2.3 | 0.9727 | 2.2 |
| 50 | 50 | 0.9265 | 2.8 | 0.9391 | 3.1 | 0.9419 | 3.3 | 0.9688 | 1.7 | 0.9636 | 1.6 |
| 100 | 100 | 0.9349 | 1.4 | 0.9418 | 1.5 | 0.9429 | 1.5 | 0.9603 | 1.2 | 0.9574 | 1.2 |
| 125 | 125 | 0.9361 | 1.6 | 0.9419 | 1.3 | 0.9428 | 1.3 | 0.9578 | 1.1 | 0.9550 | 1.0 |
| 150 | 150 | 0.9374 | 1.1 | 0.9424 | 1.1 | 0.9432 | 1.2 | 0.9561 | 1.0 | 0.9537 | 1.0 |
| 200 | 200 | 0.9427 | 0.9 | 0.9459 | 0.9 | 0.9465 | 0.9 | 0.9571 | 0.8 | 0.9555 | 0.8 |
| 250 | 250 | 0.9417 | 0.8 | 0.9444 | 0.8 | 0.9446 | 0.8 | 0.9537 | 0.8 | 0.9524 | 0.7 |
| 300 | 300 | 0.9425 | 0.7 | 0.9448 | 0.7 | 0.9451 | 0.7 | 0.9532 | 0.7 | 0.9519 | 0.7 |
| 10 | 20 | 0.8745 | 19.9 | 0.9051 | 37.0 | 0.9253 | 50.2 | 0.9720 | 3.3 | 0.9591 | 2.9 |
| 25 | 50 | 0.9119 | 5.4 | 0.9295 | 6.8 | 0.9345 | 7.2 | 0.9624 | 2.0 | 0.9546 | 1.9 |
| 50 | 100 | 0.9252 | 2.0 | 0.9358 | 2.5 | 0.9382 | 2.9 | 0.9575 | 1.4 | 0.9522 | 1.4 |

| | | t(10) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n1 | n2 | **Cps(U1)** | **Aws(U1)** | **Cps(M1)** | **Aws(M1)** | **Cps(M2)** | **Aws(M2)** | **Cps(U2)** | **Aws(U2)** | **Cps(M3)** | **Aws(M3)** |
| 100 | 200 | 0.9354 | 1.2 | 0.9405 | 1.2 | 0.9416 | 1.2 | 0.9550 | 1.0 | 0.9525 | 1.0 |
| 125 | 250 | 0.9378 | 1.0 | 0.9423 | 1.0 | 0.9431 | 1.0 | 0.9536 | 0.9 | 0.9516 | 0.9 |
| 150 | 300 | 0.9409 | 0.9 | 0.9446 | 0.9 | 0.9449 | 0.9 | 0.9542 | 0.8 | 0.9523 | 0.8 |
| 200 | 400 | 0.9434 | 0.8 | 0.9462 | 0.8 | 0.9467 | 0.8 | 0.9540 | 0.7 | 0.9526 | 0.7 |
| 250 | 500 | 0.9418 | 0.7 | 0.9441 | 0.7 | 0.9445 | 0.7 | 0.9519 | 0.7 | 0.9506 | 0.6 |
| 300 | 600 | 0.9424 | 0.6 | 0.9442 | 0.6 | 0.9445 | 0.6 | 0.9505 | 0.6 | 0.9498 | 0.6 |
| 20 | 10 | 0.8773 | 22.5 | 0.9084 | 28.5 | 0.9285 | 44.8 | 0.9705 | 3.3 | 0.9581 | 2.9 |
| 50 | 25 | 0.9142 | 5.0 | 0.9320 | 6.7 | 0.9368 | 8.0 | 0.9625 | 2.0 | 0.9545 | 1.9 |
| 100 | 50 | 0.9267 | 2.1 | 0.9368 | 2.4 | 0.9392 | 2.6 | 0.9603 | 1.4 | 0.9555 | 1.4 |
| 200 | 100 | 0.9343 | 1.2 | 0.9399 | 1.2 | 0.9410 | 1.2 | 0.9547 | 1.0 | 0.9520 | 1.0 |
| 250 | 125 | 0.9382 | 1.0 | 0.9425 | 1.0 | 0.9432 | 1.0 | 0.9538 | 0.9 | 0.9520 | 0.9 |
| 300 | 150 | 0.9395 | 0.9 | 0.9430 | 0.9 | 0.9439 | 0.9 | 0.9545 | 0.8 | 0.9526 | 0.8 |
| 400 | 200 | 0.9395 | 0.8 | 0.9422 | 0.8 | 0.9428 | 0.8 | 0.9514 | 0.7 | 0.9500 | 0.7 |
| 500 | 250 | 0.9420 | 0.7 | 0.9442 | 0.7 | 0.9446 | 0.7 | 0.9520 | 0.7 | 0.9511 | 0.6 |
| 600 | 300 | 0.9427 | 0.6 | 0.9450 | 0.6 | 0.9454 | 0.6 | 0.9512 | 0.6 | 0.9503 | 0.6 |

TABLE 2: Simulated 95% coverage probabilities (Cps) and average widths (Aws) of the U1, M1, M2, U2 and M3 for normal distributions with various sample sizes of balanced and unbalanced designs in which the variances are equal and unequal respectively.

| equal vars | Normal | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **n1,n2** | **Cps (u1)** | **Aws(U 1)** | **Cps (M1)** | **Aws (M1)** | **Cps(M2)** | **Aws (M2)** | **Cps (U2)** | **Aws (U2)** | **Cps (M3)** | **Aws (M3)** |
| 10,10 | 0.8884 | 17.05 | 0.9170 | 28.01 | 0.9398 | 42.74 | 0.9969 | 2.74 | 0.9908 | 2.46 |
| 25,25 | 0.9308 | 2.39 | 0.9461 | 3.01 | 0.9507 | 3.57 | 0.9762 | 1.60 | 0.9677 | 1.54 |
| 50,50 | 0.9423 | 1.33 | 0.9504 | 1.41 | 0.9526 | 1.42 | 0.9635 | 1.12 | 0.9586 | 1.10 |
| 100,100 | 0.9466 | 0.86 | 0.9511 | 0.88 | 0.9521 | 0.89 | 0.9561 | 0.79 | 0.9540 | 0.78 |
| 125,125 | 0.9459 | 0.75 | 0.9501 | 0.77 | 0.9508 | 0.77 | 0.9545 | 0.70 | 0.9528 | 0.70 |
| 150,150 | 0.9489 | 0.68 | 0.9521 | 0.69 | 0.9527 | 0.69 | 0.9550 | 0.64 | 0.9537 | 0.64 |
| 200,200 | 0.9489 | 0.58 | 0.9518 | 0.59 | 0.9522 | 0.59 | 0.9531 | 0.55 | 0.9520 | 0.55 |
| 250,250 | 0.9484 | 0.51 | 0.9507 | 0.52 | 0.9510 | 0.52 | 0.9528 | 0.50 | 0.9517 | 0.49 |
| 300,300 | 0.9473 | 0.47 | 0.9492 | 0.47 | 0.9494 | 0.47 | 0.9506 | 0.45 | 0.9501 | 0.45 |
| 10,20 | 0.8999 | 11.32 | 0.9234 | 20.14 | 0.9392 | 21.70 | 0.9653 | 2.29 | 0.9537 | 2.10 |
| 25,50 | 0.9310 | 1.88 | 0.9430 | 2.18 | 0.9466 | 2.48 | 0.9574 | 1.38 | 0.9515 | 1.34 |
| 50,100 | 0.9427 | 1.10 | 0.9496 | 1.15 | 0.9510 | 1.16 | 0.9557 | 0.97 | 0.9521 | 0.95 |
| 100,200 | 0.9455 | 0.73 | 0.9491 | 0.74 | 0.9500 | 0.74 | 0.9521 | 0.68 | 0.9502 | 0.67 |
| 125,250 | 0.9459 | 0.64 | 0.9486 | 0.65 | 0.9490 | 0.65 | 0.9523 | 0.61 | 0.9508 | 0.60 |
| 150,300 | 0.9463 | 0.58 | 0.9485 | 0.59 | 0.9490 | 0.59 | 0.9511 | 0.55 | 0.9495 | 0.55 |
| 200,400 | 0.9465 | 0.50 | 0.9483 | 0.50 | 0.9486 | 0.50 | 0.9496 | 0.48 | 0.9487 | 0.48 |
| 250,500 | 0.9488 | 0.44 | 0.9502 | 0.45 | 0.9504 | 0.45 | 0.9515 | 0.43 | 0.9507 | 0.43 |
| 300,600 | 0.9478 | 0.40 | 0.9490 | 0.40 | 0.9491 | 0.40 | 0.9502 | 0.39 | 0.9496 | 0.39 |
| equal vars | | | | | Normal | | | | | |
| **n1,n2** | **Cps (u1)** | **Aws (U1)** | **Cps (M1)** | **Aws (M1)** | **Cps(M2)** | **Aws (M2)** | **Cps (U2)** | **Aws (U2)** | **Cps (M3)** | **Aws (M3)** |
| 20,10 | 0.8991 | 9.32 | 0.9242 | 12.28 | 0.9396 | 27.02 | 0.9648 | 2.29 | 0.9524 | 2.10 |
| 50,25 | 0.9310 | 1.86 | 0.9429 | 2.23 | 0.9463 | 3.49 | 0.9582 | 1.38 | 0.9512 | 1.33 |
| 100,50 | 0.9416 | 1.10 | 0.9481 | 1.15 | 0.9496 | 1.16 | 0.9548 | 0.97 | 0.9514 | 0.95 |
| 200,100 | 0.9467 | 0.73 | 0.9502 | 0.74 | 0.9507 | 0.74 | 0.9526 | 0.68 | 0.9506 | 0.67 |
| 250,125 | 0.9457 | 0.64 | 0.9488 | 0.65 | 0.9494 | 0.65 | 0.9516 | 0.61 | 0.9499 | 0.60 |
| 300,150 | 0.9472 | 0.58 | 0.9496 | 0.59 | 0.9501 | 0.59 | 0.9516 | 0.55 | 0.9503 | 0.55 |
| 400,200 | 0.9454 | 0.50 | 0.9472 | 0.50 | 0.9475 | 0.50 | 0.9492 | 0.48 | 0.9483 | 0.48 |
| 500,250 | 0.9486 | 0.44 | 0.9500 | 0.45 | 0.9503 | 0.45 | 0.9513 | 0.43 | 0.9504 | 0.43 |
| 600,300 | 0.9492 | 0.40 | 0.9503 | 0.40 | 0.9504 | 0.40 | 0.9511 | 0.39 | 0.9506 | 0.39 |

| unequal vars | Normal | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **n1,n2** | **Cps (u1)** | **Aws (U1)** | **Cps (M1)** | **Aws (M1)** | **Cps(M2)** | **Aws (M2)** | **Cps (U2)** | **Aws (U2)** | **Cps (M3)** | **Aws (M3)** |
| 10,10 | 0.8854 | 12.74 | 0.9128 | 22.20 | 0.9372 | 30.98 | 0.8996 | 2.00 | 0.9204 | 2.26 |
| 25,25 | 0.9280 | 1.82 | 0.9415 | 2.30 | 0.9460 | 2.73 | 0.9281 | 1.28 | 0.9349 | 1.34 |
| 50,50 | 0.9385 | 1.02 | 0.9460 | 1.08 | 0.9479 | 1.09 | 0.9451 | 0.92 | 0.9482 | 0.95 |
| 100,100 | 0.9449 | 0.67 | 0.9486 | 0.68 | 0.9495 | 0.69 | 0.9544 | 0.66 | 0.9561 | 0.67 |
| 125,125 | 0.9443 | 0.59 | 0.9473 | 0.60 | 0.9478 | 0.60 | 0.9564 | 0.59 | 0.9579 | 0.60 |
| 150,150 | 0.9472 | 0.53 | 0.9500 | 0.54 | 0.9506 | 0.54 | 0.9588 | 0.54 | 0.9604 | 0.55 |
| 200,200 | 0.9468 | 0.46 | 0.9491 | 0.46 | 0.9496 | 0.46 | 0.9602 | 0.47 | 0.9616 | 0.47 |
| 250,250 | 0.9467 | 0.40 | 0.9482 | 0.41 | 0.9484 | 0.41 | 0.9613 | 0.42 | 0.9620 | 0.42 |
| 300,300 | 0.9467 | 0.37 | 0.9478 | 0.37 | 0.9479 | 0.37 | 0.9617 | 0.38 | 0.9622 | 0.39 |
| 10,20 | 0.8791 | 10.38 | 0.9369 | 7.41 | 0.9239 | 20.19 | 0.8672 | 1.90 | 0.8840 | 2.14 |
| 25,50 | 0.9191 | 1.61 | 0.9501 | 1.47 | 0.9341 | 2.17 | 0.9149 | 1.23 | 0.9205 | 1.29 |
| 50,100 | 0.9359 | 0.94 | 0.9518 | 0.80 | 0.9431 | 0.99 | 0.9390 | 0.89 | 0.9418 | 0.91 |
| 100,200 | 0.9407 | 0.62 | 0.9523 | 0.52 | 0.9441 | 0.64 | 0.9507 | 0.63 | 0.9523 | 0.64 |
| 125,250 | 0.9426 | 0.55 | 0.9510 | 0.46 | 0.9451 | 0.56 | 0.9549 | 0.57 | 0.9561 | 0.57 |
| 150,300 | 0.9428 | 0.50 | 0.9505 | 0.41 | 0.9450 | 0.51 | 0.9561 | 0.52 | 0.9571 | 0.52 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 200,400 | 0.9442 | 0.43 | 0.9506 | 0.35 | 0.9457 | 0.43 | 0.9580 | 0.45 | 0.9589 | 0.45 |
| 250,500 | 0.9456 | 0.38 | 0.9499 | 0.31 | 0.9468 | 0.38 | 0.9615 | 0.40 | 0.9623 | 0.41 |
| 300,600 | 0.9472 | 0.35 | 0.9503 | 0.28 | 0.9485 | 0.35 | 0.9612 | 0.37 | 0.9618 | 0.37 |
| 20,10 | 0.9162 | 5.63 | 0.9031 | 18.83 | 0.9495 | 15.25 | 0.9524 | 1.54 | 0.9658 | 1.66 |
| 50,25 | 0.9398 | 1.26 | 0.9296 | 1.87 | 0.9533 | 2.10 | 0.9552 | 0.99 | 0.9610 | 1.02 |
| 100,50 | 0.9458 | 0.77 | 0.9415 | 0.97 | 0.9529 | 0.80 | 0.9581 | 0.71 | 0.9608 | 0.72 |
| 200,100 | 0.9489 | 0.51 | 0.9436 | 0.63 | 0.9529 | 0.52 | 0.9599 | 0.50 | 0.9615 | 0.51 |
| 250,125 | 0.9483 | 0.45 | 0.9445 | 0.56 | 0.9514 | 0.46 | 0.9605 | 0.45 | 0.9618 | 0.45 |
| 300,150 | 0.9480 | 0.41 | 0.9445 | 0.50 | 0.9508 | 0.41 | 0.9602 | 0.41 | 0.9612 | 0.41 |
| 400,200 | 0.9489 | 0.35 | 0.9454 | 0.43 | 0.9509 | 0.35 | 0.9603 | 0.36 | 0.9611 | 0.36 |
| 500,250 | 0.9488 | 0.31 | 0.9465 | 0.38 | 0.9500 | 0.31 | 0.9605 | 0.32 | 0.9609 | 0.32 |
| 600,300 | 0.9493 | 0.28 | 0.9483 | 0.35 | 0.9504 | 0.28 | 0.9608 | 0.29 | 0.9613 | 0.29 |

From Table2, when samples are drawn from an identical normal distribution, the U1 is clearly the poorest performer for all situations and all cases regardless of balance or unbalance designs and is the widest interval whereas the U2 usually has coverage a little bit above nominal that higher than others. For moderate to large sample sizes, the M1, M2 and M3 generally produce almost identical results that are quite close to the target level for moderate to large sample sizes and become almost in distinguishable when sample sizes are large but the M3 might be preferable since it produces a little bit shorter interval widths on average. It should be note, however, that the coverage of U1 and U2 are also converge to the nominal level as the sample sizes increase.

When the samples were come from normal distribution but with unequal population variances (and of cause, they both have the same kurtosis by default) the results demonstrate that the intervals that generated from the first method (i.e., the U1,M1 and M2) give not only better coverage than those of the second method (i.e.,U2 and M3) but also converge to the target level as sample sizes are moderate or large regardless of balance or unbalance designs while the confidence limits generated from the U2 and M3 usually yield coverage probabilities that are exceed the nominal coverage level as the variance difference increases when samples are moderate or large in both equal group sizes or unequal group sizes. Moreover, the M2 is still identical to the M1 and both are performed well in terms of maintaining their coverage as samples are moderate or large in sizes.

In addition, for all confidence intervals investigation when samples are drawn from a variety of nonnormal distributions in which the variances are not equal has already been considered but does not report in our study because they provided badly results, for example, they sometimes show the extremely large departure from the nominal level or give too few values of coverage probabilities quite often. This is such the important evidence that the samples which are drawn from any nonnormal distributions that are not identically distributed cannot be used to construct any of the intervals investigated in this study.

## 4. CONCLUSIONS

It is found that the three interval estimations, the M1, M2 and M3 which based up on the MBBE of variance are all better perform than those which based up on the usual unbiased sample variance estimators, the U1and U2 even when samples are drawn from normal distributions. It is also found that the M2 is not only outperform than others for skewed distributions but also holds its level well for normal and symmetric nonnormal distributions while the M3 is out perform than others for normal and symmetric nonnormal distributions but is usually liberal for skewed distribution. Thus, with logically reasonable in generating confidence intervals by its theoretically extension from the MOVER method we then recommended using M2 as an alternative hybrid confidence interval estimation for variance difference at this time, we make a notice on our experimental that in generating confidence intervals, we did not split up the groups of observations into sub groups but carried out an analysis of the two population variance differences. Therefore, with the given of unequal population variances, the value of kurtosis then change and can correspond to different distribution shapes, to make a comparison between some pairs of dissimilar distributions are then not appropriated in this present study. To avoid the suggested problem, we left for the researchers as a further study.

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